Math 10B with Professor Stankova
Worksheet, Discussion #18; Tuesday, 3/20/2018
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1 Recursion Equations

1.1 Concepts

1. A first order linear recurrence equation is of the form $a_n = \alpha a_{n-1} + \beta$. The general solution to this is
$$a_n = \alpha^n a_0 + \beta\frac{\alpha^n - 1}{\alpha - 1}$$
if $\alpha \neq 1$ and $a_n = a_0 + n\beta$ otherwise.

1.2 Examples

2. Solve the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$.

2 Differential Equations

2.1 Concepts

3. A problem of the form $y' = f(t, y)$ and $y(0) = y_0$ is called an **initial value problem (IVP)**. There is a theorem that tells us when a solution to this problem exists. It says that if f is continuous, then for every choice of y_0, the solution **exists** in a time interval $[0, T)$ for some $0 < T \leq \infty$. But, the solution may not exist everywhere and it is not guaranteed to be unique.

   To solve a linear first order differential equation, first bring y, y' to one side and then divide to get the coefficient of y' to be 1 so you have something of the form $y' + P(x)y = Q(x)$. Then, multiply the integrating factor $I(x) = e^{\int P(x)dx}$. We define it this way because then $I' = I(x)P(x)$ (convince yourself of this). This gives $I(x)y' + I(x)P(x)y = I(x)Q(x)$. Then the left side is just $(I(x)y)' = I(x)Q(x)$ and we can solve this by integrating then dividing by $I(x)$.

   Another form of differential equation that has a nice solution is the case of separable equations. A differential equation is separable if we can write it as $y' = f(y)g(t)$, a term only involving ys and constants with a term only involving ts and constants. To do this, we write $y' = \frac{dy}{dt}$ and move all the ts to one side and the ys to the other to get $\frac{dy}{f(y)} = g(t)dt$. Now integrating gives us a solution. Often, we do not explicitly solve for y and leave them in an implicit form.

2.2 Examples

4. Find the general solution to $y' - \frac{y}{x+1} = (x+1)^2$.

5. Find the solution to $y'e^y = 2t + 1$ with $y(1) = 0$.

2.3 Problems

6. True False We cannot use the method of separable equations on $y' = e^{y+t}$ because it involves a sum of y and t.

7. True False If we can use the method of separable equations, we must be able to write $y' = (ay + b)f(t)$ for a linear polynomial in terms of y.

8. True False The equation $y' = y + t$ is not separable and so we do not know how to solve it.

9. Find the solution of $y' + \frac{y}{x} = e^x/x$ with $y(1) = 0$.

10. Find the solution of $y' + 2xy = 2x$ with $y(0) = 0$.

11. Find the solution to $r' = r^2/t$ with $r(1) = 1$.

12. Find the general solution of $y' = 2t \sec y$.

2.4 Extra Problems

13. Find the solution to $\frac{dy}{dt} = 3t^2y^3 + e^ty^3$ with $y(0) = -1$.

14. Find the solution to $\dfrac{dy}{dx} = 6xy^2$ with $y(1) = 1/4$.

15. Find the solution to $\dfrac{dy}{dx} = \frac{3x^2+2x+1}{2y+1}$ with $y(0) = 1$.

16. Find the solution to $\dfrac{dy}{dt} = 2y + 3$ with $y(0) = 0$.

17. Find the solution to $\dfrac{dx}{dy} = e^{x-y}$ with $x(0) = 0$.